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A Supplement to 「A Note on Algebras and Semiprime Rings」

Ryo SAITO*

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Let R be a commutative ring. In [3], we consider an R-algebra A which satisfies the following condition.

(*) For any R-algebra B such that B is a semiprime ring, $B\otimes_R A$ is also a semiprime ring.

Proposition. Let A be an R-algebra which is finitely generated as an R-module and satisfies the condition (*). Then A is a separable R-algebra.

Proof. For any maximal ideal $\mathfrak{m}$ of R, $A\otimes_R R/\mathfrak{m}$ is a semiprime ring and a finite dimensional $R/\mathfrak{m}$-algebra which satisfies the condition (*). For any extension field L of $R/\mathfrak{m}$, since $A\otimes_R R/\mathfrak{m}\otimes_{R/\mathfrak{m}} L$ is isomorphic to $A\otimes_R L$ and $\mathrm{rad}(A\otimes_R L)=0$, $A\otimes_R R/\mathfrak{m}$ is a separable $R/\mathfrak{m}$-algebra. Hence A is a separable R-algebra ([1], p. 72, Th. 7.1).

A is said to be an H-separable R-algebra if A is a separable C-algebra and the map $C\otimes_R C\to C$ $(x\otimes y\to xy)$ is an isomorphism where C is the center of A ([4], p. 265, Prop. 1.1). A central separable algebra is an H-separable algebra and satisfies the condition (*) ([3], p. 394, Prop. 7). But regarding the relation between an H-separable algebra and an algebra which satisfies the condition (*), we can see that an H-separable algebra does not necessarily satisfy the condition (*) and an algebra which satisfies the condition (*) is not necessarily H-separable.

Examples.

(1) Let Z be the ring of rational integers. $Z/(n^k)$ $(n>0,\ k\geqq 2)$ is not semiprime and this is an H-separable Z-algebra.

(2) Let L be a separable extension field of a field K such that $[L:K]<\infty$. Then K-algebra L satisfies the condition (*) ([2], p. 164, Lemma 7). But since the map $L\otimes_K L\to L$ $(x\otimes y\to xy)$ is not isomorphic, L is not H-separable.

* 酪農科，数学研究室　斎藤　暸
Hokkaido College of Arts and Sciences (Mathematics), Ebetsu, Hokkaido 069, Japan

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4) Sugano, K., 1967. Note on semisimple extensions and separable extensions. Osaka J. Math. 4: 265-270.

要　　約

前論文 [3] の補足として，条件 (*) を満たす有限生成多元環は分離的であることと，条件 (*) を満たす多元環と H-分離多元環との関係について述べた。